GEORGIA INSTITUTE OF TECHNOLOGY

ENGINEERING EXPERIMENT STATION

ATLANTA, GEORGIA 30332

February 5, 1965

Studebaker Corporation
CTL Division
P. O. Box 15227
Cincinnati 15, Ohio

Attn: Mr. C. F. Zapf
Project Engineer

Subject: Monthly Progress Letter 1, Project A-836
"Prediction of the Mechanical/Elastic Properties of Epoxy Matrices Reinforced with Silicon Carbide Platelets"
Sub-Contract to Contract AF 33(615)-2112
Covering the Period January 15 to January 31, 1965

Gentlemen:

An analysis of the structural behavior of epoxy matrices reinforced with silicon carbide platelets is being conducted. The objective of this analysis is to provide a rational basis for the prediction and optimization of such a composite.

In this analysis, the composite will be treated as a two-phase material with the major surfaces of the platelets in parallel alignment. The orientation of the platelets will be considered random in all planes normal to the plane of alignment.

Smith and Guttman 1/ carried out a topological analysis of the two-dimensional section generated by the random sectioning of a three-dimensional structure. During the period covered by this report, a detailed study was made of this topological analysis with the object of developing a geometric basis for prediction of the properties of platelet reinforced composites. The study produced results of great importance to the task of property prediction and also indicated methods of structure analysis which may be ideally suited to future correlations with experimental results.

Based on the Smith-Guttman approach, the volume fraction of platelets was rigorously proved to be identical to the section area fraction of platelets. The section area fraction represents the effective load bearing area of the platelets which would lie on a plane passed through the composite at random, normal to the direction of applied stress. If this section area is assumed to be under constant stress or constant strain, the strength of the composite may be calculated by weighing the strengths of platelet and matrix on a

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1/Smith, Cyril S., and Leoter Guttman, "Measurement of Internal Boundaries in Three-Dimensional Structures by Random Sectioning," J. Metals 5 81-87 (1953).

volume fraction basis as a result of the proved equality of the volume and section area fraction. The combination of the law of mixtures with the results of the topological analysis thus permits a prediction of the strength of a reinforced composite. Attention should be given to the fact that this analysis involves no assumptions as to the platelet shape and is valid for random orientation of the platelets about the axes normal to the applied stress.

The topological analysis also produced a technique for rapid determination of the platelet surface-to-volume ratio, thickness, mean breadth, volume fraction, and section area fraction from a single micrograph of the sectioned composite. With little additional work, the analysis can be extended to prediction of the directional strength in an anisotropic composite in which the platelets are parallel but selectively oriented rather than randomly oriented. This situation could arise in certain methods of fabrication if the platelets are produced by chopping rather than by random fracture. These applications of the topological analysis are outside the scope of the present work but they may prove highly useful in future work.

The technique described above will permit first-stage prediction of the composite strength. However, this initial prediction will be subject to the assumption of constant stress throughout the platelet. Since this condition may not be met in an actual composite, consideration will be given to possible modifications of the strength prediction to account for non-uniform platelet stress.

In addition to predictions of composite strength, work during the coming month will be concerned with developing relationships between the independent elastic coefficients of a platelet composite and the elastic and geometric parameters of the constituent materials. The governing constitutive equation of the composite will be taken as the generalized Hooke's Law. The composite will be assumed to be macroscopically homogeneous. That is, stain and stress averages taken over large enough sub-regions of the specimen will be the same for any such subregion. The symmetry of the composite will be taken to be two-dimensional orthotropy on the basis of the platelet packing arrangement previously stated.

Respectfully submitted,

William J. Corbett
Project Director

WJCjm

GEORGIA INSTITUTE OF TECHNOLOGY

ENGINEERING EXPERIMENT STATION

ATLANTA, GEORGIA 30332

March 8, 1965

Studebaker Corporation
CTL Division
P. O. Box 15227
Cincinnati, Ohio 45215

Attention: Mr. C. F. Zapf
Project Engineer

Subject: Monthly Progress Letter 2, Project A-836
"Prediction of the Mechanical/Elastic Properties of Epoxy Matrices Reinforced with Silicon Carbide Platelets"
Sub-Contract to Contract AF 33(615)-2112
Covering the Period February 1 to February 28, 1965

Gentlemen:

The strength data for SiC flake, obtained by Carborundum and reported in the January Letter Report, were subjected to a statistical analysis. There appeared to be no significant correlation between the flake thickness and strength. All flakes tested were, therfore, considered to be randomly selected replicates, and the average strength at the 95% confidence level, was found to be $(4.44 \pm 1.09) \times 10^5$ psi.

The results of the statistical analysis were also used to give at least a rough estimate of the measured average strength, based on 48 randomly selected flakes, which may be expected in any other lot of flakes produced and tested in the same manner. This analysis assumes that no systematic differences exist from lot to lot in the flake production technique or from specimen to specimen in the strength test. Under the assumed conditions of the analysis, the average lot strength may be expected to fall between 2.9×10^5 and 6.0×10^5 psi at the 95% level of confidence and between 2.4×10^5 and 6.5×10^5 psi at the 99% level of confidence. In the absence of more representative measurements in the future, these values will be used as indications of the strength of the reinforcement for computing theoretical predictions of composite strength.

As reported in the January Letter Report, a rigorous analysis has shown that, for a platelet reinforced composite where the platelets are in parallel alignment, the volume fraction of platelets is equal to the section area fraction of platelets. The section area fraction is the effective load bearing area of the platelets which would lie on a plane passed through the composite at random, normal to the direction of applied stress. This relationship has been used during the period covered by this report to establish the relationships describing the ultimate tensile strength of the composite if the failure mechanism is that of platelet fracture. In establishing the theoretical strength relationship for this condition the following assumptions were made:

1. That stress is proportional to strain in both materials up to the point at which the reinforcement fails.

2. The platelets and matrix are securely bonded together so they are strained an equal amount under load.

3. The platelets are straight and in parallel alignment in the direction of applied load.

4. The composite is macroscopically homogeneous and isotropic in all directions within the plane of platelet alignment.

From assumption 1,

$$\sigma = E\epsilon \tag{1}$$

where σ is stress, E is modulus of elasticity, and ϵ is strain. From assumption 2

$$\epsilon_c = \epsilon_p = \epsilon_m \tag{2}$$

where the subscripts c, p, and m denote the composite, platelets and matrix respectively. Since load is proportional to unit stress times area, the load on the platelets is

$$W_p = \sigma_p A_p = E_p \epsilon_c A_p, \tag{3}$$

and the load on the matrix is

$$W_m = \sigma_m A_m = E_m \epsilon_c A_m, \tag{4}$$

where W is the load and A is the section area fraction. For a unit area of composite the following relationship can be written for the composite stress

$$\sigma_c = E_p \epsilon_c X_p + E_m \epsilon_c (1 - X_p) \tag{5}$$

where X_p is the volume fraction which, it can be shown, is equal to the section area fraction. At the point of platelet fracture the strain in the platelet will be equal to the ultimate strain for this material, ϵ_p^*, and from equation (2) this will be equal to the strain throughout the composite. Therefore, the relationship for the ultimate tensile strength of the composite σ_c^* for the condition of failure by platelet fracture is

$$\sigma_c^* = E_p \epsilon_p^* X_p + E_m \epsilon_p^* (1 - X_p) \tag{6}$$

The above relationship is valid for all practical concentrations of platelet reinforcement. However, at very low platelet concentrations the equation becomes invalid because the stress on the matrix may not exceed its ultimate tensile strength even after it assumes the load originally carried by the platelets. That is, if

$$\sigma_m^* (1 - X_p) > E_p \epsilon_p^* X_p + E_m \epsilon_p^* (1 - X_p) \tag{7}$$

where σ_m^* is the ultimate tensile strength of the matrix, failure will not occur. Therefore for values of composite strength equal to or less than the ultimate tensile strength of the matrix the strength of the composite is given by

$$\sigma_c^* = \sigma_m^* (1 - X_p) \tag{8}$$

and it can be shown that the platelet concentration, $(X_p)_{crit.}$, below which equation (8) controls is

$$(X_p)_{crit.} = \left[1 + \left(\frac{\sigma_m^*}{\sigma_p^*} - \frac{E_m}{E_p} \right)^{-1} \right]^{-1} \tag{9}$$

It is also worth noting that there exists a value of platelet concentration, X_p', below which the presence of the platelets actually serves to weaken the matrix rather than reinforce it. This value is obtained by solving equation (6) for the condition where the ultimate tensile strength of the composite σ_c^* is equal to the ultimate tensile strength of the matrix material

$$\sigma_c^* = \sigma_m^* = E_p \epsilon_p^* X_p' + E_m \epsilon_p^* (1 - X_p') \qquad (10)$$

and is found to be

$$X_p' = \frac{1}{E_p - E_m} \left(\frac{\sigma_m^*}{\sigma_p^*} - E_m \right) \qquad (11)$$

There are, of course, other mechanisms by which a composite produced from discontinuous reinforcements, such as platelets, can fail. These are:

1. Fracture of the matrix due to insufficient platelet overlap at low platelet concentrations.

2. Shear failure in the matrix at the ends of the platelets.

3. Failure of the bond between the platelets and the matrix.

Efforts during the coming month will be made to evaluate the composite strength based on other failure mechanisms as well as continuing the effort toward predicting the elastic properties of the composite.

Respectfully submitted,

William J. Corbett
Project Director

GEORGIA INSTITUTE OF TECHNOLOGY

ENGINEERING EXPERIMENT STATION
ATLANTA, GEORGIA 30332

April 9, 1965

Studebaker Corporation
CTL Division
P. O. Box 15227
Cincinnati, Ohio 45215

Attention: Mr. C. F. Zapf
Project Engineer

Subject: Monthly Progress Letter 3, Project A-836
"Prediction of the Mechanical/Elastic Properties of Epoxy Matrices Reinforced With Silicon Carbide Platelets"
Sub-Contract to Contract AF 33(615)-2112
Covering the Period March 1 to March 31, 1965

Gentlemen:

Efforts were made to classify a sample of silicon carbon platelets, as supplied by The Carborundum Company, according to size and shape using a water elutriation column. A rough classification according to size was achieved, but no classification by shape was achieved. The fraction containing the smaller platelets contained small twins and clumps, and the fractions containing the larger platelets contained large twins and clumps.

An analysis has been conducted to estimate the strength of a platelet reinforced epoxy matrix assuming the failure mechanism to be shear failure in the matrix. The analysis is similar to that used by Rosen, Dow, and Hashin 1/ for fiber reinforced composites. A three component model consisting of a platelet surrounded by a thin layer of matrix contained in a body of composite material has been used, and a shear lag type analysis has been employed to estimate the stresses. The analysis is very nearly completed. However, typical values have not yet been used to estimate the composite strength based on this failure mechanism. When completed, this analysis and the analysis based upon the failure mechanism of platelet fracture are expected to establish the most probable failure mechanism and indicate the strength level that can be expected with the

1/B. W. Rosen, N. F. Dow and E. Hashin, "Mechanical Properties of Fibrous Composites," NASA Contractor Report NASA CR-31 (1964).

platelet reinforced composites. Also the present analysis should serve to determine the direction of desired improvement in matrix characteristics to obtain a stronger composite.

During the coming month the analysis based on the shear failure mechanism will be completed. Composite strengths will be estimated using this analysis and the one previously completed for a reinforcement failure mechanism. The transverse and longitudinal elastic moduli will also be estimated.

Respectfully submitted,

William J. Corbett
Project Director

WJC/jw

FINAL SUMMARY REPORT

Project No. A-836

PREDICTION OF THE MECHANICAL AND ELASTIC PROPERTIES OF EPOXY MATRICES REINFORCED WITH SILICON CARBIDE PLATELETS

By

WILLIAM J. CORBETT and J. D. FLEMING, JR.

CONTRACT NO. GT-64-11
CTL DIVISION
STUDEBAKER CORPORATION

15 JANUARY 1965 TO 15 JULY 1965

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia

FINAL SUMMARY REPORT

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SUMMARY

The objective of this study was the prediction of the mechanical and elastic properties of a composite material consisting of an epoxy matrix reinforced with silicon carbide platelets.

A topological analysis was performed to prove, rigorously, that the section area fractions of the constituents of a platelet, reinforced composite on any plane passed at a random position through the composite, could be determined from a knowledge of only the volume fractions of the constituents. It was shown that the section area fractions of the constituents were equal to the volume fractions for a totally random orientation, or for platelets oriented with their major axes parallel. This characterization of platelet reinforced composites was necessary to the estimation of their mechanical and elastic properties from just the knowledge of the properties and relative quantities of their constituents. That this analysis represents a very powerful analytical technique for the complete, geometrical charactization of composites from simple experimental measurements, has also been pointed out.

The longitudinal and transverse elastic moduli of the composites were estimated from the results of a bounding technique based on the variation theorems of the theory of elasticity. These elastic moduli, for a platelet reinforced composite, appear to coincide with the upper and lower bounds, respectively. Plots are presented for the variation of both the longitudinal and transverse elastic moduli with platelet concentration for an Epon 828, epoxy matrix reinforced with silicon carbide platelets. Estimates are also presented for the effect of variations in the elastic modulus of the matrix on the longitudinal and transverse elastic moduli of composites reinforced with silicon carbide platelets.

The ultimate strengths of composites, consisting of an Epon 828, epoxy matrix reinforced with silicon carbide platelets, were predicted using two possible mechanisms of composite failure - platelet fracture and shear failure of the matrix in the neighborhood of the platelet ends. Plots of the estimated, ultimate strength, with varying platelet concentration, are presented for both of these failure mechanisms. On the basis of these estimated strengths, it is predicted that the failure mechanism for the Epon 828, silicon carbide platelet composites, will be by shear failure of the matrix in the neighborhood of the platelet ends, and the ultimate strengths will be in the neighborhood of 10^4 psi. It is noted that the available experimental data appear to confirm these analytical conclusions.

Recommendations are presented for future studies to refine the estimates of the mechanical and elastic properties of platelet reinforced composites.

I. INTRODUCTION

This study was performed as a subcontract with the CTL Division of Studebaker Corporation under an Air Materials Laboratory contract (AF 33(615)-2112) for research on structural composites. The objective of this study was the prediction of the mechanical and elastic properties of a composite material consisting of an epoxy matrix reinforced with silicon carbide platelets.

There were two principle reasons for conducting this study. First, it was felt that these estimates of the mechanical and elastic properties would be very useful in evaluating the results of an experimental study conducted by CTL on epoxy matrices reinforced with oriented, silicon carbide platelets. Also, it was felt that this study could provide estimates of the maximum potential of such composites, and indicate the direction of future efforts to achieve this potential.

II. TOPOLOGICAL ANALYSIS

Prediction of the behavior of a flake reinforced composite under stress requires geometric characterization of a load plane generated by passing a plane through the composite normal to the direction of stress. While several approaches to this problem were available, the analytical technique chosen was that of random sectioning. This type of topological analysis showed promise not only of providing the necessary load plane characterization, but also of serving as a basis for experimental analysis of the composite structure in future work.

The pioneer work of Smith and Guttman 1/ showed that the intersection of a plane with an irregular solid may be represented by the intersection of a number of equivalent planes with the cutting plane. Integration of the total perimeter of the intersection for all possible orientations of the plane elements shows that, for a solid particle intersected by a plane,

$$\frac{p}{A} = \frac{\pi}{4} \quad \frac{S}{v} \tag{1}$$

where

p = perimeter of the intersection

A = area of the intersection

S = surface area of the particle

v = volume of the particle,

as shown in Figure 1.

1/Smith, Cyril S. and Lester Guttman, "Measurement of Internal Boundaries in Three-Dimensional Structures by Random Sectioning," J. Metals 5, 81-7 (1953).

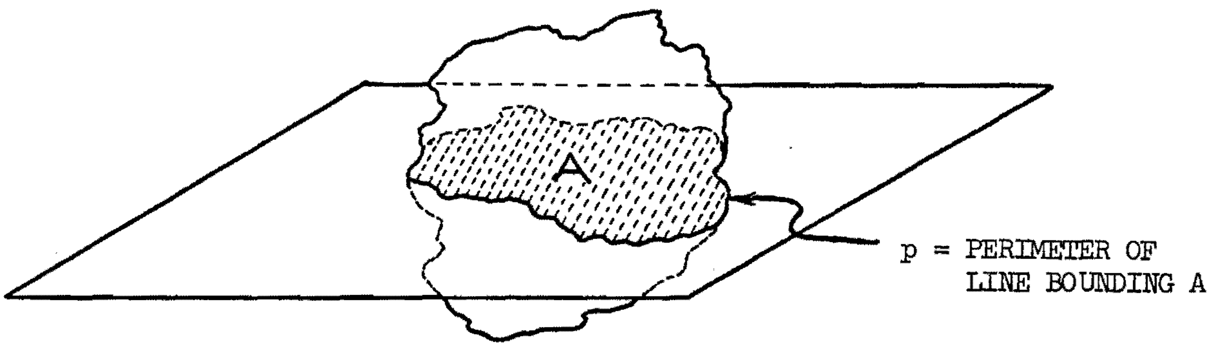

Figure 1. Intersection of a Plane With a Solid Particle.

If a curved plane intersects a family of grid lines with a spacing, d, the number of intersections is given, by integration over all possible grid orientations, by

$$N = \frac{2\ell}{\pi d} \tag{2}$$

where

N = number of intersections of grid lines with intersection line

ℓ = length of intersection line

d = spacing of grid lines.

Since any surface may be represented by a series of curved planes, Equation 2 also represents the length of the intersection line bounding the area A in Figure 1. This intersection line is, of course, the perimeter, p.

If the grid spacing is small or if the grid is positioned across the area A many times at random,

$$A = Ld \tag{3}$$

where

L = total of the grid line lengths lying within the area A

d = grid line spacing,

and substitution into Equation 2 and solution for the total length of the intersection line, p, yields

$$p = \frac{\pi}{2} \frac{NA}{L} \tag{4}$$

Figure 2 demonstrates the meaning of the terms in Equation 4.

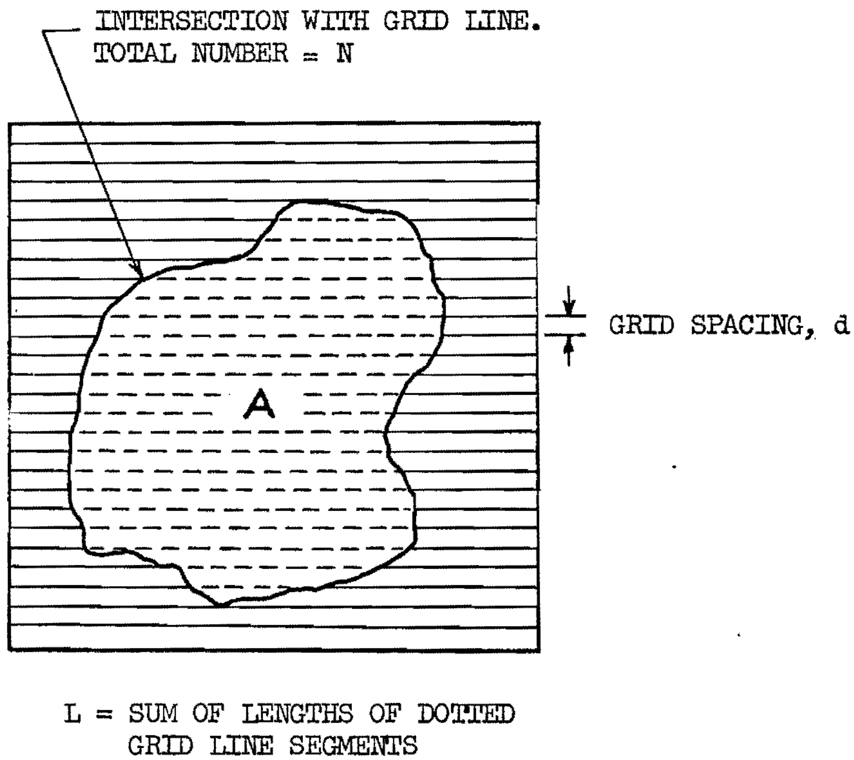

Figure 2. Model Demonstrating the Determination of Grid Line Intersections With a Particle.

Combining Equations 1 and 4 yields

$$\frac{S}{v} = 2\,\frac{N}{L} \tag{5}$$

Equation 5 involves no assumption as to the particle shape or orientation. It is based only on the assumption that average values of N and L may be obtained and this is possible with repeated application of the grid at various random orientations. If the system consists of many particles, the average value of S/v may be obtained by repeated application of the grid to the system in many different locations without changing its orientation. In practice, the particular technique used to obtain average values of N and L may be chosen for convenience, the only requirement being that of a truly random measurement.

If a plane is passed through a composite* and a grid imposed on the plane, the total number of intersections of grid lines with particle boundaries is related to the average number of intersections per particle by

$$N = \frac{N_t}{n} \tag{6}$$

where

N = average number of intersections per particle.

N_t = total number of intersections with particles cut by the plane.

n = number of particles cut by the plane.

*In the treatment of composites, early definition of terms is helpful. In this discussion, "section area" designates the area of the given phase lying on a plane passing through the composite. The section area for the particles would be the sum of all areas depicted by the area A in Figure 1. The section area fraction of a phase represents the portion of the total area of the intersecting plane which is section area of that phase. Surface area has its usual connotation, i.e., the area of the boundaries separating the phase under consideration from all other phases.

In the same manner,

$$L = \frac{L_p}{n} \tag{7}$$

where

L = average grid line length within particle section area.

L_p = total grid line length within particle section area.

Since, from Equation 5,

$$\frac{S}{v} = 2 \frac{N}{L},$$

$$\frac{S}{v} = 2 \frac{\frac{N_t}{n}}{\frac{L_p}{n}}$$

or

$$\frac{nS}{nv} = 2 \frac{N_t}{L_p}$$

or

$$S_p = 2 \frac{N_t}{L_p} v_p \tag{8}$$

where

S_p = total surface area of particles cut by the plane

v_p = total volume of particles cut by the plane.

In the same manner,

$$S_m = 2 \frac{N_t}{L_m} v_m \tag{9}$$

where

S_m = total surface area of matrix cut by the plane

N_t = total number of intersections of grid lines with matrix boundaries

L_m = total length of grid line within matrix section area

v_m = volume of matrix associated with surface area S_m.

It is apparent that S_p and S_m both represent the interfacial area between the particles cut by the plane and the matrix associated with these particles. In addition, each intersection of a grid line with a particle boundary also represents an intersection with a matrix boundary so that N_t is the same in Equations 8 and 9. Combining these two equations,

$$\frac{v_m}{L_m} = \frac{v_p}{L_p}$$

or

$$\frac{v_p}{v_m} = \frac{L_p}{L_m} \tag{10}$$

The volume fraction, V_p, of the particles is thus

$$V_p = \frac{v_p}{1 + v_m} = \frac{\frac{L_p}{L_m}}{1 + \frac{L_p}{L_m}} = \frac{L_p}{L_m + L_p}$$

and since the total length of grid lines, L_t, is

$$V_p = \frac{L_p}{L_t} \tag{11}$$

The total section area, A_p, of the particles is given by Equation 3

$$A_p = L_p d. \tag{12}$$

The total area covered by the grid, A_t, is

$$A_t = L_t d. \tag{13}$$

Combining Equations 12 and 13,

$$\frac{A_p}{A_t} = \frac{L_p}{L_t} .$$

The section area fraction of the particles, A_p, is thus

$$A_p = \frac{L_p}{L_t} . \tag{14}$$

Combining Equations 11 and 14 shows that

$$V_p = A_p. \tag{15}$$

Equation 15 shows that the section area fraction of the particles equals the particle volume fraction. The section area is the same as the particle load bearing area on any plane passed at a random position through the composite normal to the direction of an applied stress.

III. ESTIMATION OF ELASTIC MODULI

The variational, or energy, theorems of the theory of elasticity have been used by a number of investigators 2,3,4,5,6/ in attempts to predict the elastic moduli of composite materials. However, an examination of most of the existing works reveals that they are principally modifications of the method first introduced by Paul 2/. In this early work, Paul derived bounds for the elastic modulus, or Young's modulus, of a two-phase material having arbitrary phase geometry. These bounds were obtained, using the variational theorems of the theory of elasticity, by taking the same simple tension in the matrix and the reinforcement as an admissible stress system, and a simple tension deformation for an admissible displacement field.

Working with a unit cube of the composite, and assuming a tensile stress to be applied to two opposite faces with zero stress on the lateral surface, Paul found the lower bound of the elastic modulus to be given by the expression

$$E_c \geq \frac{1}{\frac{1-V_p}{E_m} + \frac{V_p}{E_p}} \tag{16}$$

2/Paul, B., "Prediction of Elastic Constants of Multiphase Materials," Trans. AIME 218, 36-41 (1960).

3/Hashin, Zvi, "The Elastic Moduli of Heterogeneous Materials," J. Appl. Mech. 29, 143-150 (1962).

4/Hill, R., "Elastic Properties of Reinforced Solids: Some Theoretical Principles," J. Mech. Phys. Solids 11, 357-372 (1963).

5/Hashin, Zvi and B. W. Rosen, "The Elastic Moduli of Fiber-Reinforced Materials," J. Appl. Mech. 31, 223-232 (1964).

6/Tsai, S. W., "Structural Behavior of Composite Materials," NASA Contractor Report, NASA CR-71 (1964).

where

E_c = elastic modulus of the composite

E_m = elastic modulus of the matrix

E_p = elastic modulus of the reinforcement

and

V_p = volume fraction of the reinforcement.

Also, from the tensile strain induced in the specimen, Paul found the upper bound on the elastic modulus to be expressed by

$$E_c \le \frac{1 - \nu_m + 2C\,(C - 2\nu_m)}{1 - \nu_m - 2\nu_m^2}\; E_m\,(1 - V_p)$$

$$+ \frac{1 - \nu_p + 2C\,(C - 2\nu_p)}{1 - \nu_p - 2\nu_p^2}\; E_p\,(V_p) \qquad (17)$$

where

ν_p = Poisson's ratio for reinforcement

ν_m = Poisson's ratio for matrix

and C is a factor given by the expression

$$C = \frac{\nu_m\,(1 + \nu_p)\,(1 - 2\nu_p)\,(1 - V_p)E_m + \nu_p\,(1 + \nu_m)\,(1 - 2\nu_m)\,V_p\,E_p}{(1 + \nu_p)\,(1 - 2\nu_p)\,(1 - V_p)E_m + (1 + \nu_m)\,(1 - 2\nu_m)\,V_p\,E_p}$$

These bounds are quite general, being independent of the geometry of the reinforcement. These bounds are also quite exact, and the value of the elastic modulus of any two phase composite should lie somewhere between the values predicted by these bounds*. Unfortunately, however, when applied to a composite that is macroscopically isotropic (i.e., random orientation of the reinforcement), the range of values contained by these bounds is generally so great as to preclude a useful estimate of the elastic modulus. Many of the subsequent analyses, using the variational theorems, have been directed toward narrowing this range by the use of various approximations.

In spite of its apparent shortcomings for macroscopically isotropic composites, it appears, upon careful examination, that Paul's bounding technique can provide very good estimates of one or more of the elastic moduli for macroscopically anisotropic composites (i.e., uniform orientation of the reinforcement). This is because one or more of the elastic moduli of such composites may coincide with one or more of Paul's bounds. For example, in an estimate of the longitudinal elastic modulus of unidirectional fiber reinforced composites, Hashin and Rosen 5/ obtained a result that coincides with the upper bound of Paul. As a matter of fact, the conditions of derivation of this upper bound were such that the longitudinal elastic modulus of any composite having a unidirectional orientation of reinforcement is coincident with this upper bound. That is, the reinforcement can be a prism of any cross section, and so long as the reinforcement is aligned with respect to its long axis (which is assumed to extend the entire length of the unit volume), Paul's upper bound becomes a very good estimate of the composite's longitudinal elastic modulus.

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*This analysis does not consider the possibility of the operation of any matrix restraint that might be induced by the reinforcement. Such a phenomena would probably produce a composite modulus that would exceed the upper bound.

The situation with respect to the transverse elastic modulus of unidirectional-oriented, prismatic reinforcement is not, in general, so convenient. That is, the transverse elastic modulus may not coincide with Paul's lower bound. It depends on the nature of the transverse cross-section. It does not appear, for example, that the transverse elastic modulus of a unidirectional-oriented fiber reinforced composite will coincide with Paul's lower bound unless the fibers are contiguous (i.e., in contact) along their long axis. However, it can be shown that where the transverse cross-section of the reinforcement is a slab of uniform thickness, which can be taken as passing through the unit volume, the transverse modulus of the composite will coincide with Paul's lower bound.

For a platelet reinforced composite in which the platelets have their major surfaces in parallel alignment, as can be seen from the preceding discussion, the longitudinal elastic modulus should correspond to Paul's upper bound and the transverse elastic modulus (i.e., normal to the planes of platelet alignment) should correspond to the lower bound. Therefore, taking the elastic modulus of the silicon carbide platelets to be 70×10^6 psi, as reported by the Carborundum Company, and the elastic modulus of Epon 828 to be 3.6×10^5 psi, as reported by the Shell Chemical Company, the elastic moduli of a silicon carbide-Epon 828 composite were calculated as a function of platelet volume fraction. The Poisson's ratio of the silicon carbide platelets was taken as 0.17 and the Poisson's ratio of Epon 828 as 0.40. The longitudinal elastic modulus was taken as being equal to the right side of relation (17) and the transverse modulus was taken as being equal to the right-hand side of relation (16). These data are presented in Figure 3. Since the orientation of the plates has been considered to be random within the plane of platelet alignment,

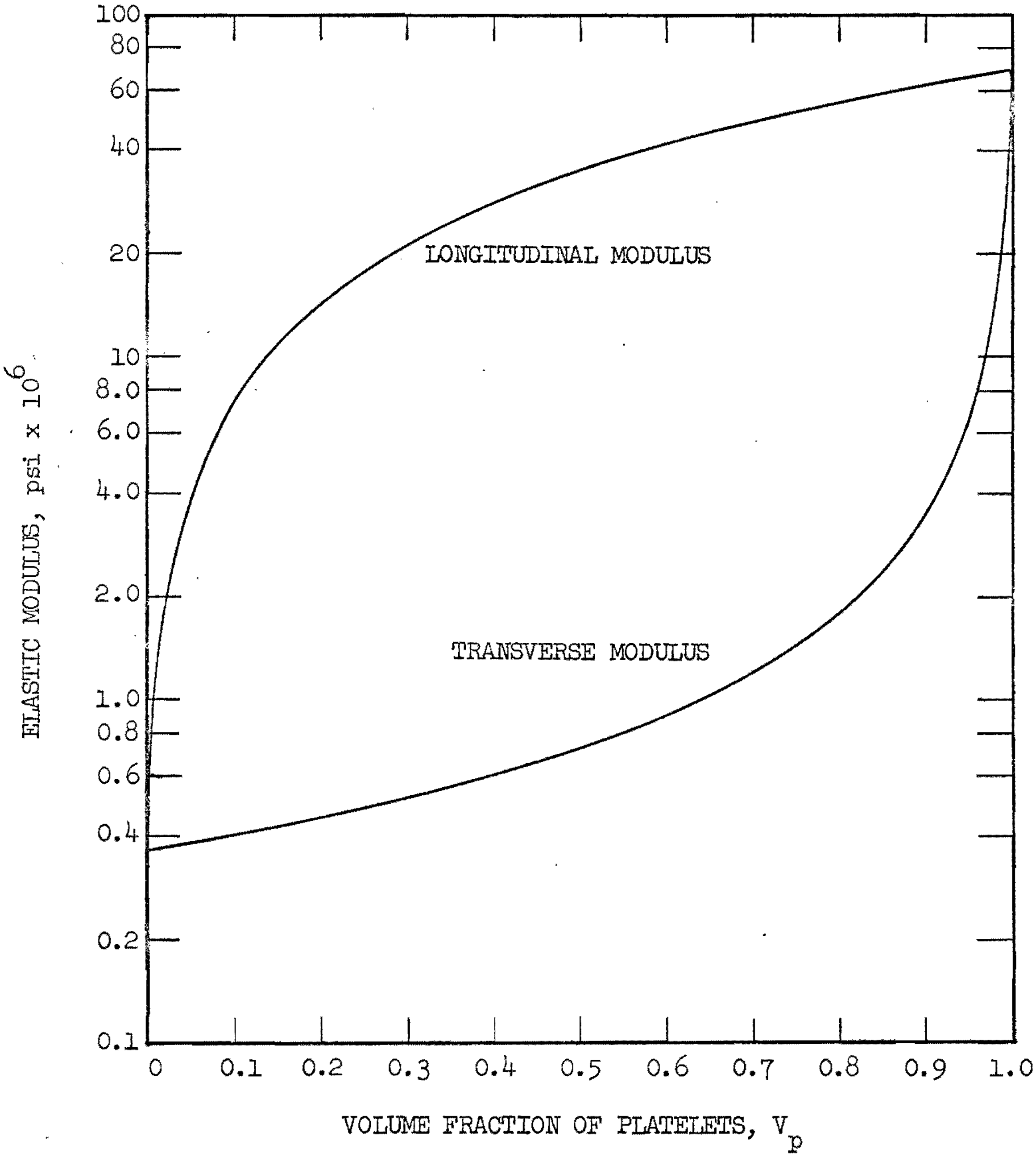

Figure 3. Elastic Modulus vs. Platelet Concentration for An Epon 828 - Silicon Carbide Platelet Composite.

the longitudinal elastic modulus of the composite will be the same in any direction within this plane of alignment.

Estimates were also made, based on the preceding analysis, of the effect of a change in the elastic modulus of the matrix on the longitudinal and transverse moduli of composites made from silicon carbide platelets. The hypothetical values of the matrix elastic moduli were 1.0×10^6 psi and 1.5×10^6 psi. These data are presented along with the data for the Epon 828 matrix for comparison in Figures 4 and 5.

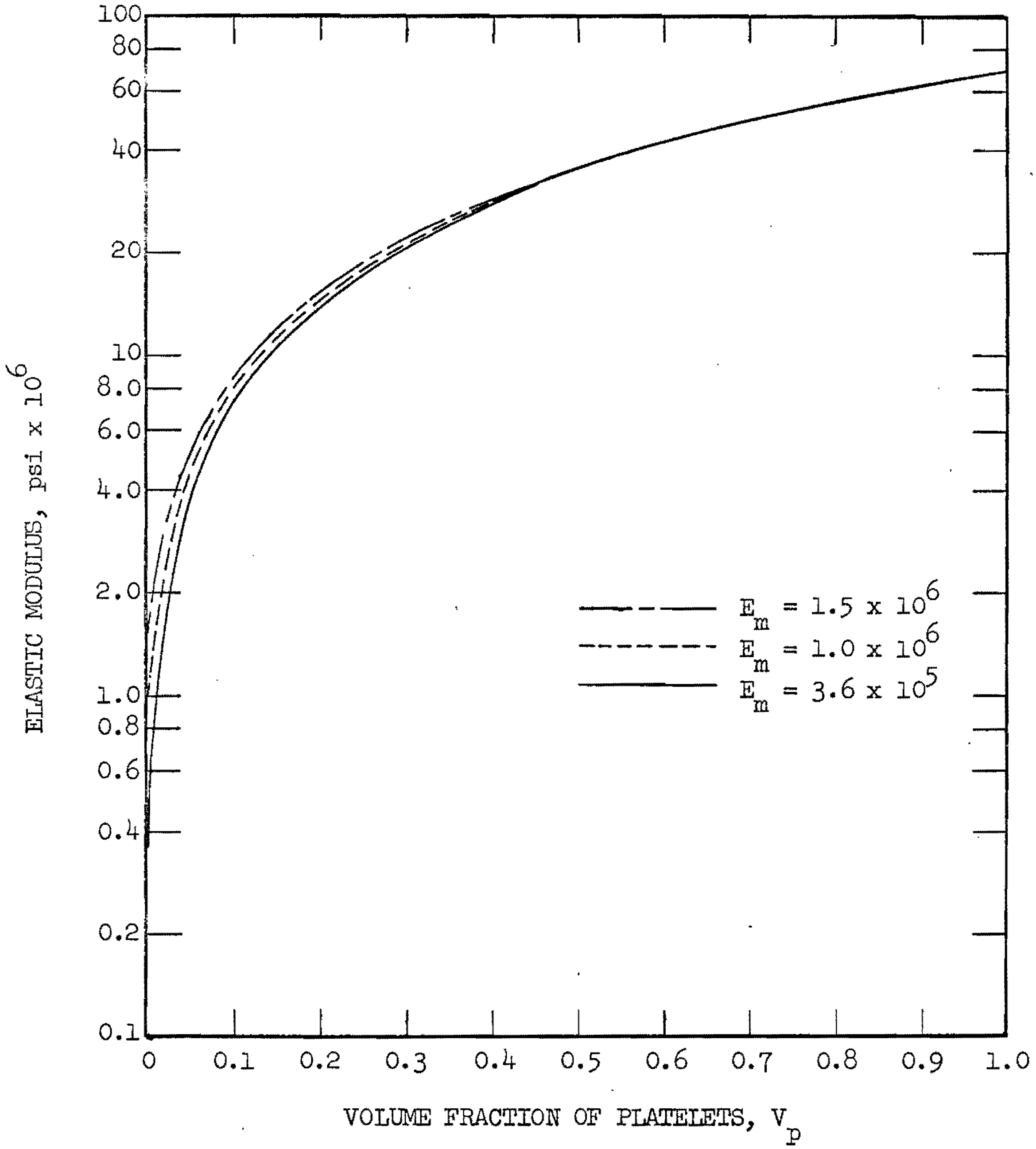

Figure 4. Longitudinal Elastic Modulus vs. Platelet Concentration as a Function of Matrix Modulus.

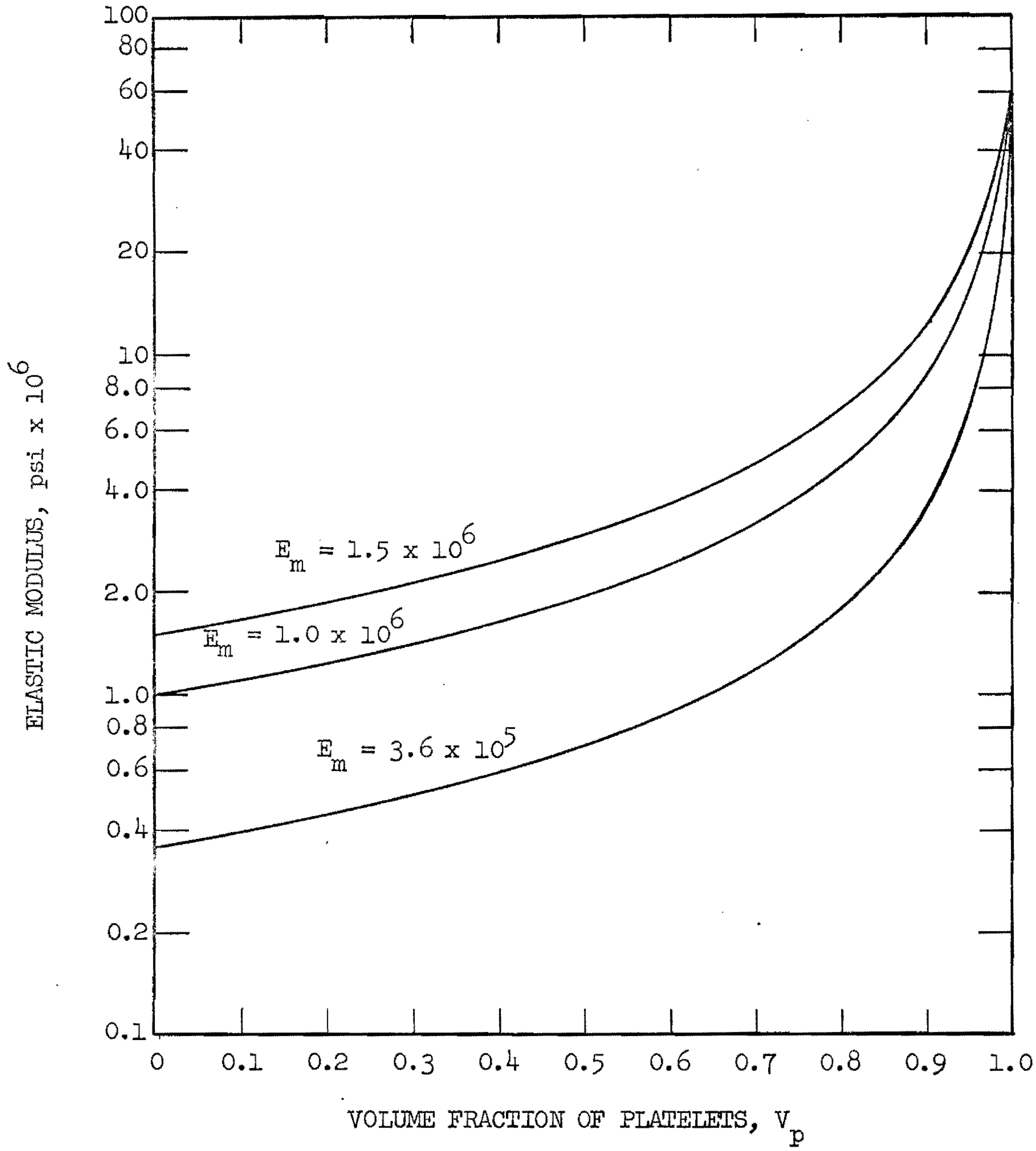

Figure 5. Transverse Elastic Modulus vs. Platelet Concentration as a Function of Matrix Modulus.

IV. ESTIMATION OF COMPOSITE STRENGTH

Any attempt to predict the strength of a composite structure must be predicated on a knowledge, or an assumption, of the mechanism of failure. For example, with a continuous form of reinforcement such as continuous fibers or ribbons, it can be reasonably assumed that failure will ultimately occur by fracture of the reinforcement. With discontinuous reinforcement, on the other hand, failure can also occur by several other mechanisms: fracture of the matrix due to insufficient reinforcement overlap; shear failure in the matrix near the ends of the platelets; or failure of the bond between the reinforcement and the matrix.

While an analysis based on the mechanism of bond failure at the matrix-platelet interface could be accomplished, it would be quite involved and, in the absence of any knowledge concerning the strength of this bond, would be of little value. Therefore, this mechanism was not considered for the present analysis. The two failure mechanisms which were considered were those of platelet fracture and shear failure of the matrix in the neighborhood of the platelet ends. For the case where failure occurs through a lack of reinforcement overlap, the strength of the composite structure will, of course, be determined by the strength of the matrix, and this is considered as a special case of the platelet fracture mechanism analysis.

1. Failure by Reinforcement Fracture

The prediction of composite strength based on reinforcement failure is the one most commonly used in estimating the properties of a composite structure, and, due to the form of the relationship obtained, it is frequently called the "law-of-mixtures technique." In estimating the theoretical composite strength based on this failure mechanism, the following assumptions were made:

1. That stress is linearly proportional to strain in both materials up to the point at which the reinforcement fails.

2. The platelets and matrix are securely bonded together so they are strained an equal amount under load.

3. The platelets are straight and in parallel alignment in the direction of applied load.

4. The composite is macroscopically homogeneous and isotropic in all directions within the plane of platelet alignment.

From assumption 1,

$$\sigma = E\epsilon \tag{18}$$

where σ is stress, E is modulus of elasticity, and ϵ is strain. From assumption 2

$$\epsilon_c = \epsilon_p = \epsilon_m \tag{19}$$

where the subscripts c, p, and m denote the composite, platelets and matrix, respectively. Since load is proportional to unit stress times area, the load on the platelets is

$$W_p = \sigma_p A_p = E_p \epsilon_c A_p, \tag{20}$$

and the load on the matrix is

$$W_m = \sigma_m A_m = E_m \epsilon_c A_m, \tag{21}$$

where W is the load and A is the section area fraction. For a unit area of composite, the following relationship can be written for the composite stress

$$\sigma_c = E_p \epsilon_c V_p + E_m \epsilon_c (1 - V_p) \tag{22}$$

where V_p is the volume fraction of the platelets which, as has been shown, is equal to the section area fraction. At the point of platelet fracture, the strain for this material, ϵ_p^* , and from Equation 19 this will be equal to the strain throughout the composite. Therefore, the relationship for the ultimate tensile strength of the composite σ_c^* for the condition of failure by platelet fracture is

$$\sigma_c^* = E_p \epsilon_p^* V_p + E_m \epsilon_p^* (1 - V_p) \ . \tag{23}$$

The above relationship is valid for all practical concentrations of platelet reinforcement. However, at very low platelet concentrations, the equation becomes invalid because the stress on the matrix may not exceed its ultimate tensile strength even after it assumes the load originally carried by the platelets. That is, if

$$\sigma_m^* (1 - V_p) > E_p \epsilon_p^* V_p + E_m \epsilon_p^* (1 - V_p) \tag{24}$$

where σ_m^* is the ultimate tensile strength of the matrix, failure will not occur. Therefore for values of composite strength equal to or less than the ultimate tensile strength of the matrix the strength of the composite is given by

$$\sigma_c^* = \sigma_m^* \; (1 - V_p) \tag{25}$$

and it can be shown that the platelet concentration, $(V_p)_{crit.}$ below, which Equation 25 controls is

$$(V_p)_{crit.} = \left[1 + \left(\frac{\sigma_m^*}{\sigma_p^*} - \frac{E_m}{E_p} \right)^{-1} \right]^{-1} \tag{26}$$

There may also exist a value of platelet concentration, V_p', below which the presence of the platelets actually serves to weaken the matrix, rather than reinforce it. This value is obtained by solving Equation 23 for the condition where the ultimate tensile strength of the composite σ_c^* is equal to the ultimate tensile strength of the matrix material

$$\sigma_c^* = \sigma_m^* = E_p \epsilon_p^* V_p' + E_m \epsilon_p^* \; (1 - V_p') \tag{27}$$

and is found to be

$$V_p' = \frac{1}{E_p - E_m} \left(\frac{\sigma_m^*}{\sigma_p^*} E_p - E_m \right) \tag{28}$$

The preceding relationships were used to obtain an estimate of the strength of a composite structure of silicon carbide platelets in an Epon 828 epoxy matrix, assuming that the strength of the composite was determined by the ultimate strength of the reinforcement. The strength data, obtained by the Carborundum Company, for the silicon carbide platelets were subjected to a statistical analysis. There appeared to be no significant correlation between platelet thickness and strength. All the platelets tested were, therefore, considered to be randomly selected replicates, and the average strength at the 95 per cent confidence level was found to be $(4.44 \pm 1.09) \times 10^5$ psi. The results of this statistical analysis were taken as being at least a rough estimate of the measured average strength which would be expected in any other lot of flakes produced and tested in the same manner. The analysis assumed that no systematic difference would exist from lot to lot in the flake production technique or from specimen to specimen in the strength test.

The modulus of elasticity of the silicon carbide platelets was obtained from the Carborundum Company and was based on their experimental findings. This value was 70×10^6 psi. The ultimate tensile strength and elastic modulus of Epon 828 containing Curing Agent Z were reported by the Shell Chemical Company as being 1.3×10^4 psi and 3.6×10^5 psi, respectively.

A plot of the ultimate strength as a function of platelet volume fraction is presented in Figure 6.

2. Failure by Matrix Shear

In a composite produced with discontinuous reinforcement, the matrix must transfer the load by shear to the adjacent fibers or platelets to enable the load to be, in effect, carried around the discontinuity. Unfortunately, the stresses in the vicinity of the discontinuity do not readily lend themselves to analysis. This has resulted in a variety of approximate treatments, most of which have been conducted for cylindrical inclusions and under the assumption that radial symmetry exists.

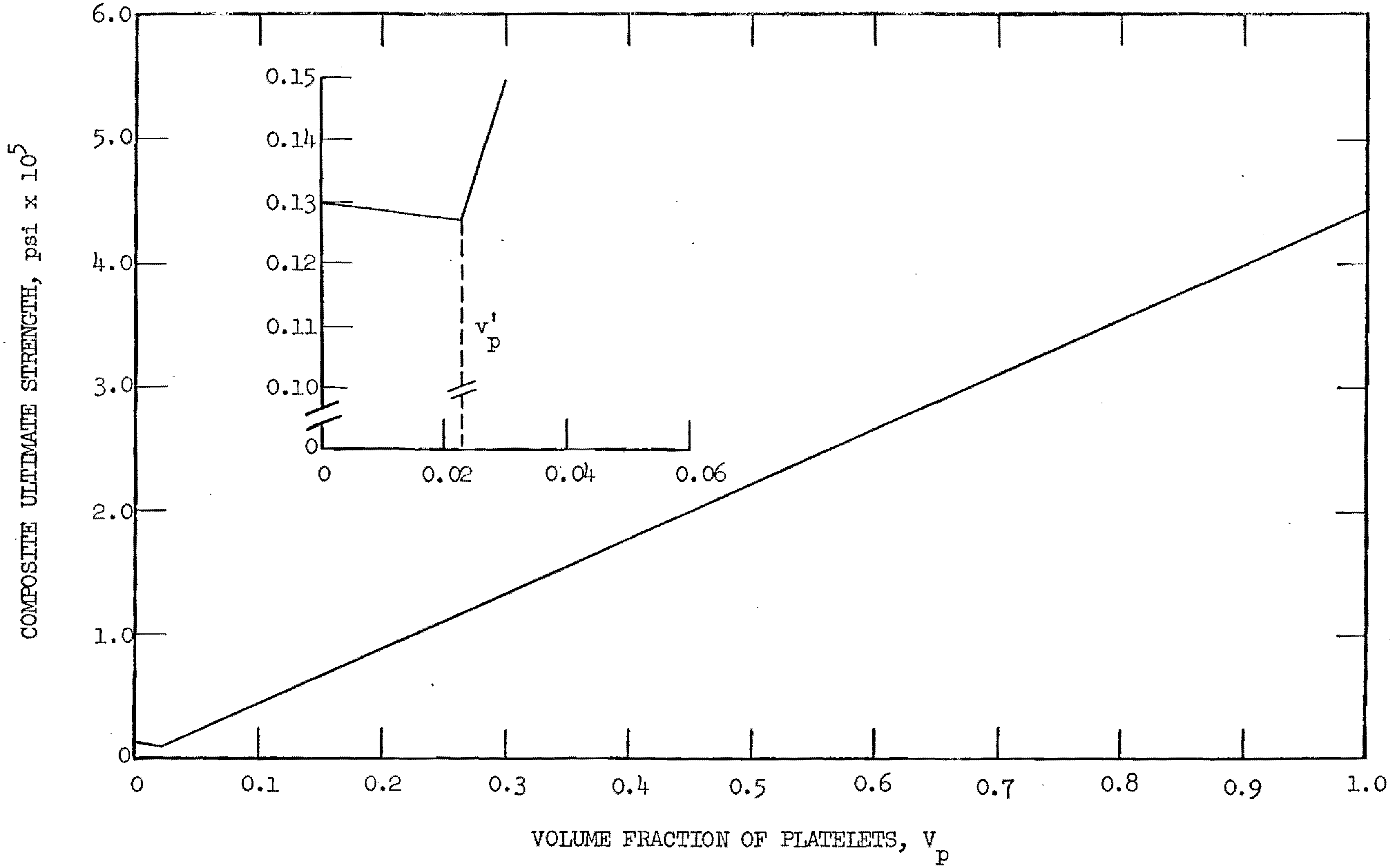

Figure 6. Composite Ultimate Strength vs. Platelet Concentration (Platelet Failure Mechanism)

Composite strengths based on shear failure of the matrix in the neighborhood of the platelet ends were estimated using a shear lag type of analysis similar to that used by Rosen, Dow and Hashin 7/ for predicting the mechanical properties of fiber reinforced composites. This analysis involves the determination of the shear stress of distribution in the matrix along the length of the platelet.

The model used is shown in Figure 7 and consists of a platelet surrounded by matrix material which, in turn, is embedded with an element of the composite material. This latter material has the average or effective properties of the composite under consideration. Load is applied to this representative volume element parallel to the direction of platelet alignment. The platelet is assumed to carry only extension stress and the matrix to transmit only shear stresses. No stress is transmitted axially from the platelet end. Shear stresses are assumed to decay in a negligible distance from the interface of the matrix with the average material. It is assumed, and can be shown to be reasonably expected (see Appendix I), that the shear stresses at the longitudinal corners of the platelet (i.e., the corners parallel to the direction of applied tensile stress) are negligible.

In the analysis, the following nomenclature will be used

$\bar{\sigma}$ = the axial stress on the entire representative volume element (RVE)

σ_p = the axial stress carried by the platelet

7/Rosen, B. W., N. F. Dow, and Z. Hashin, "Mechanical Properties of Fibrous Composites," NASA Contractor Report, NASA CR-31 (1964).

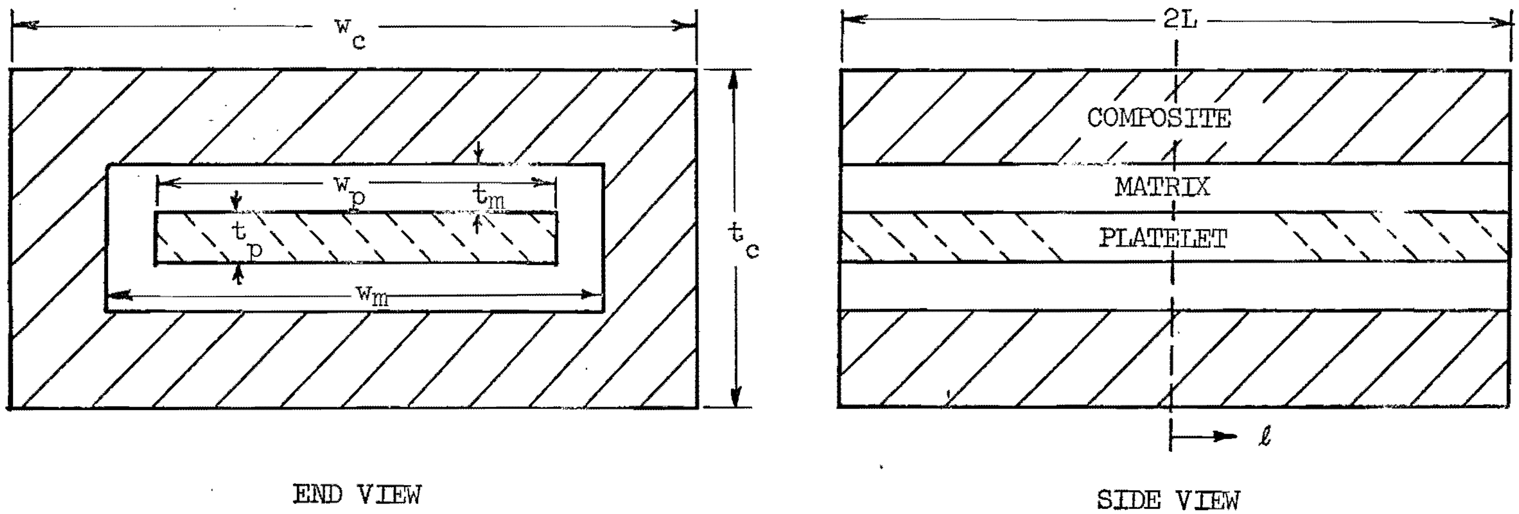

Figure 7. Model of Representative Volume Element for a Shear Lag Analysis

σ_c = the axial stress carried by the portion of the RVE that is composed of composite material

τ = the shear stress in the matrix

G_m = the elastic modulus of the matrix

γ = the shear strain in the matrix

A_ε = the area of the RVE normal to the direction of stress

A_m = the area of the matrix normal to the direction of stress

A_p = the area of the platelet normal to the direction of stress

and the subscripts m, p and c denote properties of the matrix, platelet and composite material.

It is assumed, of course, that prior to failure no displacement of the matrix relative to the platelet, or of the matrix relative to the composite material, occurs at the respective interfaces. Therefore, for equilibirum of the platelet at any point on the matrix platelet interface

$$\tau\ (2\ w_p \ell + 2\ t_p \ell) + \sigma_p w_p t_p \ = \ 0 \qquad (29)$$

or for overall platelet equilibrium

$$\tau + \tau\ (\frac{t_p}{w_p}) + \frac{t_p}{2}\ \frac{d\sigma_p}{d\ell} \ = \ 0 \qquad (30)$$

where $\frac{d\sigma_p}{d\ell}$ is the variation in the axial stress carried by the platelet with platelet length.

For equilibrium of the entire representative volume element

$$\bar{\sigma} A_\epsilon = \sigma_c (A_\epsilon - A_m - A_p) + \sigma_p A_p \quad . \tag{31}$$

Since, for any real situation A_ϵ is very nearly equal to $(A_\epsilon - A_m - A_p)$

$$\bar{\sigma} \doteq \sigma_c + \sigma_p \frac{A_p}{A_\epsilon} \quad . \tag{32}$$

Taking A_ϵ, for convenience, as being equal to a unit area and rewriting A_p in terms of the platelet width and thickness*

$$\bar{\sigma} = \sigma_c + \sigma_p (w_p t_p) \tag{33}$$

Further from the condition of no relative movement at the interfaces, the shearing strain in the matrix can be written in terms of the displacement in the composite material, U_c, and the displacement in the platelet, U_p,

$$U_c - U_p = t_m \gamma \tag{34}$$

which, in the elastic region of matrix deformation, can be written

$$U_c - U_p = \frac{t_m}{G_m} \tau \quad . \tag{35}$$

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*It must be remembered in following the analysis that the $(w_p t_p)$ term in Equation 35 will be dimensionless since it has been divided by unit area.

Differentiating Equation 35 with respect to platelet length

$$\frac{dU_c}{d\ell} - \frac{dU_p}{d\ell} = \frac{t_m}{G_m} \frac{d\tau}{d\ell} \tag{36}$$

and substituting from the stress-strain relationship

$$\frac{dU}{d\ell} = \epsilon = \frac{\sigma}{E} \tag{37}$$

yields

$$\frac{\sigma_c}{E_c} - \frac{\sigma_p}{E_p} = \frac{t_m}{G_m} \frac{d\tau}{d\ell} \quad . \tag{38}$$

Differentiating Equation 38 to obtain the expression in terms of the variation of axial stress with platelet length gives

$$\frac{1}{E_c} \frac{d\sigma_c}{d\ell} - \frac{1}{E_p} \frac{d\sigma_p}{d\ell} = \frac{t_m}{G_m} \frac{d^2\tau}{d\ell^2} \quad . \tag{39}$$

Differentiating Equation 33 and substituting for $\frac{d\sigma_c}{d\ell}$ into Equation 39 yields

$$- \frac{w_p t_p}{E_c} \frac{d\sigma_p}{d\ell} - \frac{1}{E_p} \frac{d\sigma_p}{d\ell} = \frac{t_m}{G_m} \frac{d^2\tau}{d\ell^2} \quad . \tag{40}$$

substituting for $\frac{d\sigma_p}{d\ell}$ from Equation 30 into Equation 40 gives

$$\tau \left(\frac{w_p}{E_c} + \frac{t_p}{E_c} + \frac{1}{t_p E_p} + \frac{1}{w_p E_p} \right) = \frac{t_m}{2G_m} \frac{d^2\tau}{d\ell^2} \quad . \tag{41}$$

This is of the form

$$\frac{d^2\tau}{d\ell^2} - \eta^2\tau = 0 \tag{42}$$

where

$$\eta^2 = \frac{2G_m}{t_m}\left[\frac{w_p + t_p}{E_c} + \frac{1}{E_p}\left(\frac{1}{w_p} + \frac{1}{t_p}\right)\right] \tag{43}$$

and the solution to Equation 42 is of the form

$$\tau = A \sinh \eta\ell + B \cosh \eta\ell \quad . \tag{44}$$

Taking, from the condition expressed by Equation 35, the boundary condition

$$\tau = 0 \text{ at } \ell = 0 \tag{45}$$

and imposing this boundary condition on Equation 44 yields

$$B = 0 \quad .$$

Taking as the second boundary condition

$$\sigma_p = 0 \text{ at } \ell = L \tag{46}$$

imposing this condition on Equation 38, and substituting from Equation 33 gives

$$\left.\frac{d\tau}{d\ell}\right|_{\ell = L} = \frac{\bar{\sigma}\, G_m}{E_c\, t_m} \quad . \tag{47}$$

Differentiating Equation 44 with B = 0

$$\left.\frac{d\tau}{d\ell}\right|_{\ell = L} = A\,\eta \cosh \eta\ell \tag{48}$$

and equating Equation 47 and 48 yields

$$A = \frac{\bar{\sigma}\, G_m}{\eta\, E_c t_m} \quad \frac{1}{\cosh \eta\ell} \tag{49}$$

Substituting Equation 49 into Equation 44 with B = 0 gives

$$\tau = \frac{\bar{\sigma}\, G_m}{\eta\, E_c t_m} \quad \frac{\sinh \eta\ell}{\cosh \eta L} \tag{50}$$

which may be rearranged to

$$\bar{\sigma} = \frac{\tau}{G_m} \quad \eta\, E_c t_m \quad \frac{\cosh \eta L}{\sinh \eta\ell} \tag{51}$$

Equation 51 expresses the relationship between the shearing stress developed in the matrix at a distance t_m from the platelet surface and an axial distance ℓ from the center of the platelet when an axial stress $\bar{\sigma}$ is imposed on the entire composite. In order to utilize this relationship to predict the ultimate strength of a composite, some decisions must be made concerning a critical distance for ℓ. Also, for the case of an epoxy matrix which has elastic-plastic properties, some choice of a failure condition must be made. This latter problem does not exist with a completely elastic (i.e., Hookean) matrix, since the failure condition for this type of matrix can be taken as being the ultimate shear strength of the material.

For the present case, it was reasoned that the composite material surrounding the matrix layer would probably behave elastically* and that once the matrix entered its plastic region, a strain discontinuity would develop at the matrix-composite material interface. When this occurred, failure would result. Therefore, exceeding the maximum shear yield strain of the epoxy matrix in the neighborhood of the platelet end was taken as being the criterion for failure.

Establishing the distance from the end of the platelet over which the shear strength or the yield value of the matrix must be exceeded does not have to be as arbitrary as it first appears. The limit cannot be the very end of the plate as this leads to a physically unreasonable solution. The shear must act over some finite volume element of the matrix and this requires a critical platelet length that is less than the total platelet length. An examination of the behavior of Equation 50 reveals that the increase in shearing stress along the platelet length is not linear. Rather, it rises slowly with increasing distance from the platelet center until, within the last few per cent of the length, it rises quite sharply toward its maximum value, which is, of course, attained at the end of the platelet. The highest levels of shear stress are quite localized near the platelet ends, falling off quite rapidly at a very short distance from the platelet end**. Therefore, the critical distance for failure was taken in this case to be only about 10 per cent of the length L.

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*The experimental data obtained for the silicon carbide platelet - Epon 828 composite - indicate that indeed the composite behaves elastically up to failure.

**This behavior was also pointed out by Rosen, Dow, and Hashin 7/, who found this region to be only a few fiber diameters from the fiber end.

That is, when the shearing strain in the matrix exceeded the matrix shear yield strain over only 10 per cent of the platelet half-length (about 5 platelet thicknesses in this case) failure was assumed. The value of the axial stress on the representative volume element required to produce this condition was taken as the composite ultimate strength for the case of failure by matrix shear and could be expressed as,

$$\bar{\sigma}^* = \frac{\tau_y}{G_m} \eta E_c t_m \frac{\cosh \eta L}{\sinh \eta (0.90 L)} \tag{52}$$

where.

$\bar{\sigma}^*$ = the ultimate strength of the composite

and

$\frac{\tau_y}{G_m}$ = the elastic limit of the matrix.

The elastic limit of the Epon 828 matrix was taken to be approximately 0.02, and values of the elastic modulus of the composite material were obtained from the plot of longitudinal modulus contained in Figure 3. The matrix thickness, it was found, could be expressed to a very close approximation* by

$$t_m \approx \frac{t_p}{V_p} - t_p \tag{53}$$

where V_p was the volume fraction of platelets in the composite. Composite

*Only one-half the available matrix material was assumed to be associated with any one platelet.

strengths as a function of platelet volume fraction were obtained for a composite made with platelets 500 microns in width and length and 5 microns thick (this was felt to be a good average for the platelet dimension used in most of the experimental composites). The results are presented in Figure 8.

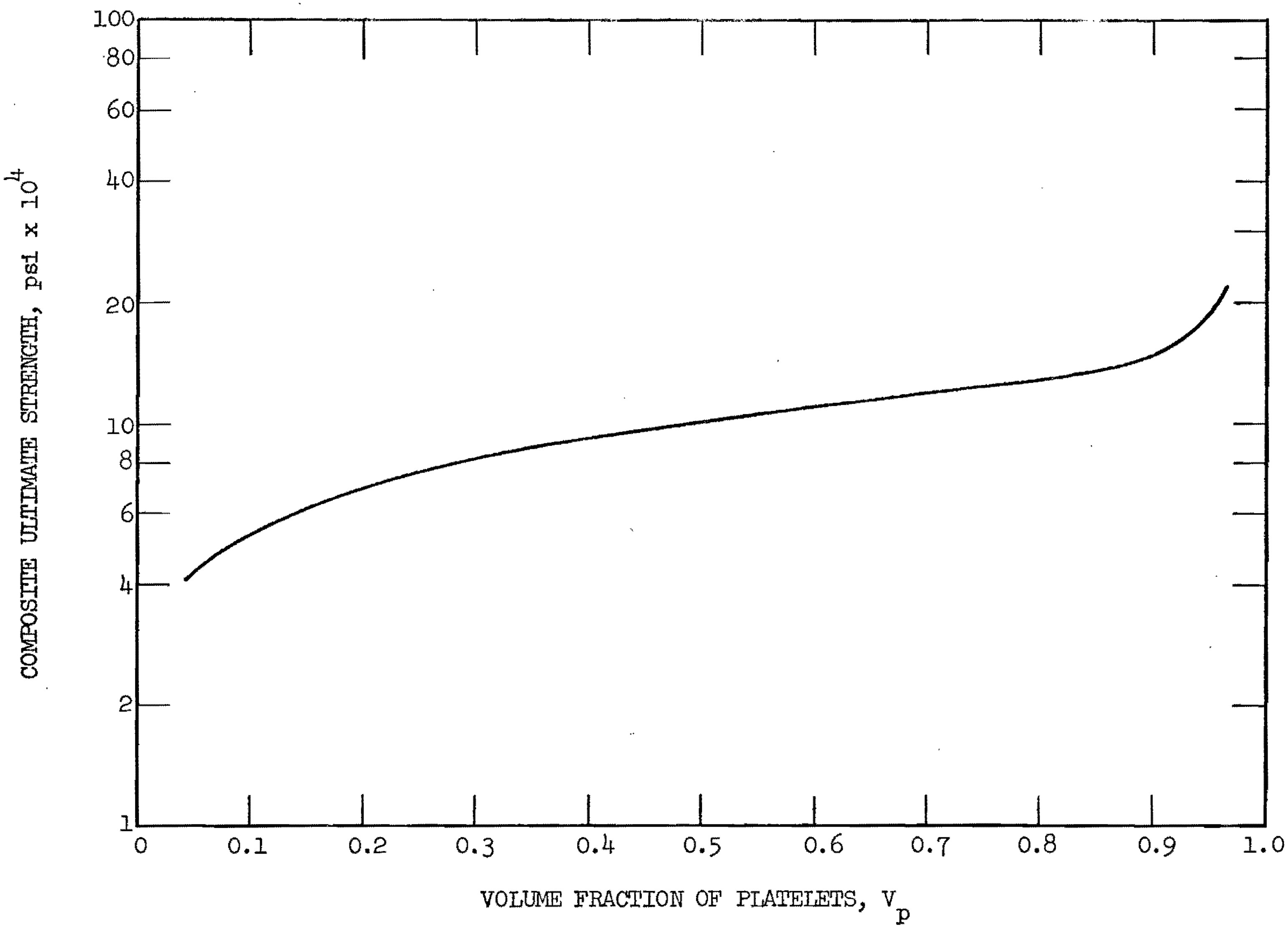

Figure 8. Composite Ultimate Strength vs. Platelet Concentration (Matrix Shear Failure Mechanism).

V. DISCUSSION OF RESULTS

The results of this effort to estimate the mechanical and elastic properties of platelet reinforced composites appear to be quite useful. Many of the experimental results for such composites, obtained by the CTL Division of Studebaker Corporation, can be better understood in view of these estimates. Also, the results of this study provide indications of the maximum potential of platelet reinforced composites, as well as indicating the direction of future efforts to achieve this potential.

The topological analysis was performed to prove, rigorously, that the section area fractions of the constituents of a platelet, reinforced composites on any plane passed at a random position through the composite, could be determined from a knowledge of only the volume fractions of the constituents. It was shown that the section area fractions of the constituents are equal to the volume fractions for a totally random orientation, or for platelets oriented with their major axes parallel. The section area is the same as the particle load-bearing area on any plane passed at a random position through the composite normal to the direction of an applied stress. This characterization of platelet reinforced composites was necessary to the estimation of their mechanical/elastic properties from just the knowledge of the properties and relative quantities of their constituents.

A rigorous proof that the section area fractions of the composite constituents are equal to the volume fractions for platelets oriented with their major axes parallel was the result sought from the topological analysis. However, it is apparent that the intermediate equations show utility as an experimental technique for analysis of composites. If a composite is sectioned and polished and a microscope grid is applied to the polished section, as described in the analysis, measurement of the grid line lengths and numbers of intersection points will permit easy experimental determination of:

a. The volume fraction of reinforcement in the composite.

b. The surface to volume ratio of the reinforcement.

c. The interfacial shear area per unit volume of the composite.

d. The average linear dimensions of the reinforcement.

e. The average reinforcement spacing.

This approach thus clearly represents a very powerful analytical technique for characterizing composites in future experimental studies.

Based on the theoretical analysis, the two most outstanding areas of performance of the silicon carbide platelet reinforced composites appear to be longitudinal elastic modulus and apparent isotropic behavior in the plane parallel to platelet alignment. Since it appears that the longitudinal elastic modulus of a platelet reinforced composite should coincide with the generalized upper bound for a two-phase composite material, the indications are that the longitudinal elastic modulus that may be obtained with a platelet reinforced composite is at least equal to that obtainable with any other geometry of discontinuous reinforcement*. While other forms of reinforcement, unidirectional oriented fibers, for example, may exhibit an equal longitudinal elastic modulus in one direction, the platelet reinforced composite should exhibit this elastic modulus in all directions in the plane of platelet alignment.

The agreement between the estimates of the longitudinal elastic moduli of composites prepared with silicon carbide platelets in a matrix of Epon 828

*The possibility of the operation of a matrix restraint mechanism was not considered in this analysis.

epoxy resin (see Figure 3) and the highest moduli obtained experimentally by CTL for such composites is quite reasonable. This provides a greater measure of confidence in the theoretically derived conclusion that longitudinal elastic modulus for platelet reinforced composites can be predicted from Paul's upper bound.

As long as the relative dimensions are such that the reinforcement maintains a plate-like configuration, both the longitudinal and transverse elastic moduli appear to be independent of platelet dimensions. There is, of course, a strong dependence on platelet concentration, with the elastic moduli increasing with increasing platelet concentration.

The estimates of the elastic moduli of hypothetical matrices with silicon carbide platelets, presented in Figures 4 and 5, are particularly interesting. It can be seen from these plots that an increase in the elastic modulus of the matrix by as much as 400 per cent will result in only a minor increase in the longitudinal elastic modulus of the composite; then only for the lower platelet concentrations. Increasing the matrix modulus produces a more significant effect on the transverse modulus of the composite. However, this property of the platelet composites should not normally be as important as the longitudinal modulus. It appears, therefore, that significant increases in the longitudinal elastic modulus of composites reinforced by silicon carbide platelets cannot reasonably be expected by using an epoxy matrix with a higher elastic modulus. The matrix elastic modulus that would be required to affect a 100 per cent increase in the longitudinal modulus of a composite containing 20-volume per cent silicon carbide flakes is about 18×10^6 psi. This is obviously beyond the range of moduli normally associated with polymeric matrices and into the range of metals. For composites containing higher fractions of silicon carbide platelets, the prospects for significant increases in the composite, longitudinal, elastic modulus, through an increase in the matrix elastic modulus, are very unattractive. For a composite containing 80-volume per cent of silicon carbide platelets, to achieve a modest 25 per cent increase in the longitudinal elastic modulus would require a matrix elastic modulus of about 150×10^6 psi. This value is beyond the range of even ceramics, except, possibly, for the case where they are in whisker form.

It appears, therefore, from the analysis of the elastic moduli of composites reinforced with silicon carbide platelets, that the most direct route to improvements in the longitudinal elastic modulus is through achieving a higher concentration of platelets in the composite, rather than using matrices with higher elastic moduli. If, however, this cannot be reasonably accomplished, a complete shift in matrix system (i.e., from polymers or resins to metals or ceramics) is indicated for obtaining significant improvements in the longitudinal elastic moduli of composites having platelet concentrations below about 35 volume per cent.

The area of least impressive performance of platelet or flake reinforced composites, both in the silicon carbide platelet composites studied by CTL and previous work 8/ with glass flake composites, has been that of tensile strength. While the platelet composite should have the advantage of strength isotropy in the plane of platelet alignment (as compared to the directional strength behavior of, say, unidirectionally oriented fiber composites), the observed strengths have not been outstanding. This lack of outstanding strength is particularly puzzling when estimates of the potential strength, based on a

8/Rugger, George, "Glass-Flake Laminates," SPE Journal 13, 35-37, 70 (1957).

reinforcement fracture mechanism, are examined (see Figure 7). However, the estimates of composite strength based on a mechanism of shear failure at the platelet ends, made in this study, show that it is this failure mechanism that controls, rather than reinforcement fracture, and that the strengths that have been observed are in the range that would be expected.

The exact values for the predicted strengths of composites of silicon carbide platelets in an Epon 828 epoxy matrix, presented in Figure 8, may not be too significant because of the estimates that were used in obtaining numerical values from the shear lag analysis. However, it is felt that these estimates were on the optimistic side (e.g., the matrix may well fail before the shear stress has exceeded the yield stress over 10 per cent of the platelet half-length), and that the predicted strengths are probably high, if anything. It is certainly felt that the order of magnitude of these values is correct. A comparison of this order of magnitude with that predicted from the reinforcement fracture mechanism (see Figure 8) shows quite definitely that the composites would be expected to fail by matrix shear, rather than reinforcement fracture. This appears to be true for any reasonable concentrations of platelets. A comparison of the values presented in Figures 8 and 9 indicate that a reversal of probable failure mechanism may occur at about 12 volume per cent platelets. However, this is considered to be rather low for practical purposes.

Further weight is given to the prediction of failure by matrix shear by the experimental results obtained at CTL. The appearance of the fracture surfaces of their composites that had been fractured in tension certainly suggests failure by matrix shear. The failure appeared to occur in the matrix around the ends of the platelets rather than through them. This was also observed in the flexure test, at least in the portion of the composite that was originally in tension. Further, the tensile strengths obtained by CTL for these composites were of the same order of magnitude (i.e., 10^4 psi) as those predicted by the shear lag analysis.

A particularly interesting feature of the strengths estimated from the shear lag analysis is the rather small increase in strength that is predicted for a very large increase in the volume fraction of platelets in the composite. This could very well explain why the tensile strengths obtained by CTL for composites of 20 to 30 volume per cent silicon carbide platelets in Epon 828 were of the same order as the results 8/ obtained for much higher loadings (55 to 60 volume per cent) of glass flakes in Epon 828. This comparison of experimental results also appears to confirm the prediction of the shear lag analysis that the strength of the composite is independent of the strength of the platelet reinforcement as long as the platelet strength is not so low that the platelet fracture mechanism would begin to control.

Unlike the case of the elastic modulus, which appears to be independent of platelet size, the shear lag analysis indicates that the composite strength is related to platelet size when the failure mechanism is matrix shear. It appears, from an examination of the relationships, that the ultimate strength of the composite will increase with an increase in length, width, and thickness of the platelets. Of course, the most direct route to improvement in composite strength appears to be an increase in the shear properties of the matrix. Since the ultimate strength of the composite is directly related to this property of the matrix, an increase in the shear yield strength (or shear strength in the case of perfectly elastic matrices) should produce a directly proportional increase in the composite strength. Consideration should also be given to the possibility of using silicon carbide whiskers for producing improvement in the shear properties of the matrix. A random dispersion of whiskers in the matrix layers might very well improve the shear properties and, thereby, allow higher composite strengths.

VI. RECOMMENDATIONS

It was not the purpose of this study to provide an exhaustive analysis of platelet reinforced composites. Rather, one of the objectives was to provide predictions for the potential performance of a particular platelet-matrix system, and to provide direction for future experimentation toward achieving, or improving, this potential. An equally important objective was to provide recommendations for further analytical studies that could lead to a better understanding of the behavior of platelet reinforced composites.

In the area of prediction of the elastic moduli, some attention should be given to the effect of platelet length on the assumption that the reinforcement extends the entire length of a composite unit volume in the longitudinal direction. There, quite probably, is some platelet length that, for all practical purposes, satisfies this condition even though the platelets are discontinuous, rather than continuous, ribbons. Conversely, there is probably some finite platelet length that is so small that it does not satisfy this condition. Also, the possibility of the operation of a matrix restraint mechanism should be investigated, particularly if future experimental results with very well characterized composites indicate that the values predicted from Paul's upper bound can be exceeded.

In the estimation of ultimate strengths of the composites, much more work needs to be done with the shear lag analysis. The establishment of criteria for failure by matrix shear should be examined in much greater depth than was possible in this study. The relationships obtained by the shear lag analysis should be programmed for electronic, digital computation, and solutions obtained for a number of conditions of platelet dimensions, composite modulus, platelet modulus, etc. A similar investigation of the shear lag failure of fiber reinforced composites should be conducted to obtain indications of whether or not one or the other reinforcement geometries is less susceptible to this mode of failure. Also, a related analysis should be performed to establish composite ultimate strength on the basis of failure of the matrix-platelet, interfacial bond. This latter analysis would, however, require input from an experimental study of the magnitude of interfacial bond strength.

APPENDIX I

Effect of Particle Radius on Matrix Stress

Exact determination of the stress field in the matrix near the sharp corner of a stressed flake would necessitate solution of the equations for six components of the stress tensor. Since this would be exceedingly complex, a study was made to see if the corner stress effects could be neglected. Dow, Rosen and Hashin 7/ showed that, in a composite, the shear stress in an elastic matrix surrounding a stressed cylindrical fiber could be represented by

$$\tau = \frac{G_b \bar{\sigma}}{\eta E_c (r_b - r_f)} \quad \frac{\sinh \eta \ell}{\cosh \eta L} \qquad (1)$$

where

G_b = bulk modulus of matrix

E_c = elastic modulus of composite

r_b = radius of matrix associated with fiber; matrix cell radius

r_f = fiber radius

$\bar{\sigma}$ = stress on composite

ℓ = axial position on fiber, measured from mid-point

L = length of fiber measured from mid-point

and

$$\eta = \frac{2\ G_b}{E_f\ (r_b - r_f)\ r_f} \tag{2}$$

where

E_f = elastic modulus of fiber.

Equation 1 shows that the matrix stress is highest at the end of the fiber, where

$$\ell = L$$

and

$$\tau_L = \frac{G_b \bar{\sigma}}{\eta\ E_c\ (r_b - r_f)} \quad \tanh\ \eta L \quad . \tag{3}$$

Since tanh $\eta\ell$ increases with increasing L, and since the limiting value of tanh is 1, the maximum matrix stress would occur if the fibers were infinitely long. This limiting stress is

$$\tau_\infty = \frac{G_b \bar{\sigma}}{\eta\ E_c\ (r_b - r_f)} \quad . \tag{4}$$

Combining Equations 1 and 3 with Equation 4 yields

$$\tau = \tau_\infty\ \frac{\sinh\ \eta\ell}{\cosh\ \eta L} \tag{5}$$

and

$$\tau_L = \tau_\infty \tanh\ \eta L \quad . \tag{6}$$

Equation 4 may be combined with Equation 2 to yield

$$\tau_{\infty} = \left[\frac{E_f \, G_b \, \bar{\sigma}^2 \, r_f}{2 \, E_c^2 \, (r_b - r_f)}\right]^{1/2} \tag{7}$$

Since r_b and r_f can be expressed in terms of the fiber volume fraction, V_f,

$$V_f = \frac{\pi \, r_f^2 \, (2 \, L)}{\pi \, r_b^2 \, (2 \, L)} = \left(\frac{r_f}{r_b}\right)^2 ,$$

equation 7 becomes

$$\tau_{\infty} = \left[\frac{E_f \, G_b \, \bar{\sigma}^2}{2 \, E_c^2 \, \left(\frac{1}{\sqrt{V_f}} - 1\right)}\right]^{1/2} \tag{8}$$

Equation 8 shows that τ_{∞} is a characterization factor for a particular fiber-matrix combination and that it is independent of the fiber radius.

Since τ_{∞} is independent of fiber radius, Equation 6 shows that the influence of fiber radius on τ_L will be determined by its influence on $\tanh \eta L$. Equation 2 shows that the influence of fiber radius on η is determined by the function

$$\Phi = (r_b - r_f) \, (r_f) \tag{9}$$

or

$$\Phi = \left(\frac{r_f}{\sqrt{V_f}} - r_f\right)(r_f) = \left(\frac{1}{\sqrt{V_f}} - 1\right) r_f^2 \tag{10}$$

If the fiber radius is allowed to change at constant volume fraction,

$$\frac{\partial \Phi}{\partial r_f} = 2\left(\frac{1}{\sqrt{V_f}} - 1\right) r_f \quad . \tag{11}$$

This shows that the function increases with r_f. As a result,

$$\downarrow r_f \equiv \uparrow \eta \tag{12}$$

Examining the influence on tanh ηL shows that

$$\frac{\partial (\tanh \eta L)}{\partial \eta} = L \operatorname{sech}^2 \eta L \geq 0 \tag{13}$$

and thus

$$\downarrow r_f \equiv \uparrow \tanh \eta L \tag{14}$$

but the limiting value of tanh ηL is one. Thus

$$\tau_L \rightarrow \tau_\infty \quad , \quad \text{as } r_f \rightarrow 0 \quad . \tag{15}$$

Also, from Equations 5 and 6,

$$\tau \leq \tau_L \quad . \tag{16}$$

From these two results, it is evident that the stress in the matrix varies as shown in Figure 9.

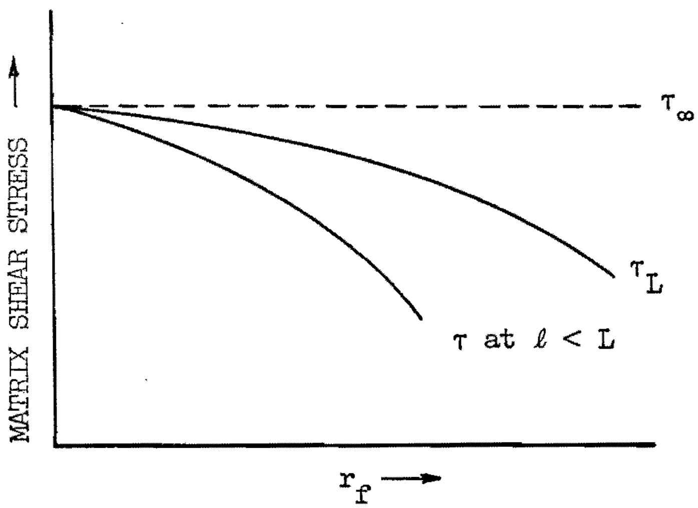

Figure 9. Variation of Matrix Shear Stress With Fiber Radius at Constant Fiber Volume Fraction.

Equation 8 represents the stress behavior as the fiber radius is decreased in a composite of constant fiber volume fraction. This condition requires that the inter-fiber spacing decrease as the radius decreases. In a flake reinforced composite, however, the corners of the flake may be approximated by comparison with fibers of zero radius but separated by a constant inter-fiber spacing. For this condition, the better approximation is to consider the variation of matrix stress with fiber radius, holding the matrix cell radius, r_b, in Equation 7 constant.

$$\tau'_\infty = \left[\frac{E_f \, G_b \, \bar{\sigma}^2 \, r_f}{2 \, E_c^2 \, (r_b - r_f)}\right]^{1/2} = \left[\frac{E_f \, G_b \, \bar{\sigma}^2}{2 \, E_c^2 \, (\frac{r_b}{r_f} - 1)}\right]^{1/2} \qquad (17)$$

where τ'_∞ represents the matrix shear stress at the end of a very long fiber in a composite of constant matrix radius, r_b. From Equation 17, it is evident that the variation of τ'_∞ with fiber radius, with all else constant, is fixed by the function

$$\psi = \frac{r_b}{r_f} - 1$$

Differentiating with respect to r_f gives

$$\frac{\partial \psi}{\partial r_f} = - \frac{r_b}{r_f^2} \qquad (18)$$

which shows that

$$\downarrow r_f \equiv \uparrow \psi \quad ,$$

thus

$$\downarrow r_f \equiv \downarrow \tau'_\infty \tag{19}$$

As before, the variation of η with r_f is set by the function

$$\Phi = (r_b - r_f)\, r_f \tag{9}$$

but now r_b is constant and

$$\frac{\partial \Phi}{\partial r_f} = r_b - 2r_f \quad . \tag{20}$$

Thus, if

$$r_b > 2r_f \quad ,$$

Φ decreases with decreasing r_f. Since r_f is to be allowed to approach zero, this condition will be met before the limit is reached and

$$\downarrow r_f \equiv \uparrow \eta \tag{21}$$

as in Equation 12. From Equation 13,

$$\downarrow r_f \equiv \uparrow \tanh \eta L \tag{22}$$

but the limiting value of tanh ηL is unity. Thus

$$\tau_L \rightarrow \tau'_\infty \quad , \text{ as } r_f \rightarrow 0 \quad . \tag{23}$$

Also

$$\tau'_\infty \rightarrow \tau_\infty \tag{24}$$

when r_f approaches the value at which

$$r_f = r_b \sqrt{V_f} \ . \tag{25}$$

Further, from Equations 5 and 6

$$\tau \leq \tau_L \tag{26}$$

at any point away from the fiber end. The behavior described by Equations 23 through 26 is shown qualitatively in Figure 10.

Figure 10 shows that the shear stress in the matrix approaches zero at the corners of a flake in a flake reinforced composite.

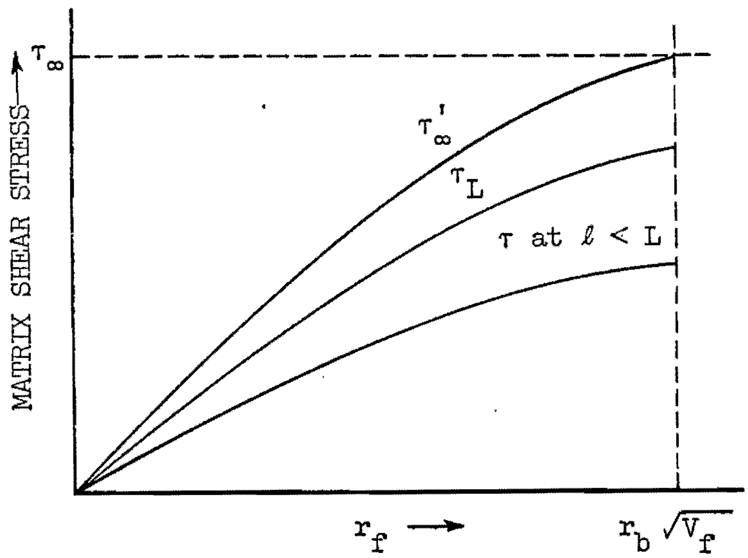

Figure 10. Variation of Matrix Shear Stress With Fiber Radius at Constant Matrix Cell Radius.